Literature Overview of Higher Order Beam Theories taking into account In-Plane and Out-of-Plane Deformation

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Abstract

The fundamental beam theories are Euler-Bernoulli and Timoshenko beam theories where the cross-section is considered undeformable either in-plane or out-of-plane. These theories provided the inspiration to the researchers in order to develop higher order beam theories, i.e. tools to reliably analyze beam members using a minimum number of equations instead of 3D or shell models. The analysis of a member employing a beam theory consists of two parts. In the first part, a cross-sectional analysis is conducted calculating the geometric constants of the cross-section of the beam. In the second part, the aforementioned geometric constants are substituted in the global equilibrium equations of the beam to calculate the response of the member under any kind of loading. The main difference among higher order beam theories and Euler-Bernoulli, Timoshenko ones is that in the first case higher order degrees of freedom are added to the model to capture warping and distortional phenomena leading to the appearance of respective higher order geometric constants. In this paper, a brief literature overview is presented regarding the major beam theories taking account out-of-plane (warping) and in-plane (distortional) deformation.

Key words: Beam theories, Distortion, In-plane deformation, Warping, Out-of-plane deformation.

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1. Introduction

In most cases in the analysis of beam-like structures, Euler-Bernoulli beam theory assumptions are adopted, while in the case of non-negligible shear deformation effect, these assumptions are relaxed by using Timoshenko beam theory. However, both theories maintain the assumptions that plane cross-sections remain plane (no out-of-plane deformation) and that their shape does not change after deformation (no in-plane deformation). In order to take into account shear lag effects in the context of a beam theory, the inclusion of non-uniform warping is necessary, relaxing the assumption of plane cross-section. In addition, the shear flow associated with non-uniform warping leads also to in-plane deformation of the cross-section, relaxing the assumption that the cross-section shape does not change after deformation. For this purpose, the so-called higher order beam theories have been developed taking into account shear lag (warping effects-out-of-plane deformation) and distortional (in-plane deformation) effects. Higher order beam theories are of increased interest due to their important advantages over approaches such as 3-D solid or shell solutions as they:

A. Require less modelling time.
B. Permit isolation of structural phenomena and results’ interpretation (rotations, warping parameters, stress resultants etc. are also evaluated in addition to displacements and stress components).
C. Facilitate modelling of supports and application of external loading.
D. Require significantly less number of degrees of freedom (dofs) reducing computational time.
E. Facilitate parametric analyses without the construction of multiple models.

Shear lag phenomenon as a consequence of non-uniform shear warping distribution in box-shaped and folded structural members has been pointed out several decades ago and many efforts have been made for its analysis e.g. by Reissner [1] employing energy method approach or by Malcolm and Redwood [2] and Moffatt and Dowling [3] using finite elements [4]. In up-to-date regulations, the significance of shear lag effects is recognized. Regarding, the provisions of European Norms with respect to shear lag effects, rules have been mainly specified in EC3-part 1.5 [5] (analysis and design of steel plated structural elements). In section 2 of EC3-part 1.5 [5], the concept of effective width for taking into account the aforementioned effect is presented. Following this concept, the effects of shear lag of flanges in global analysis may be taken into account by the use of an effective width which, for simplicity reasons, can be assumed to be uniform over the length of the span. This assumption, however, leads to an identical effect of shear lag in every point along the beam which is independent of the specific shear flow at the cross section. In Section 3 of EC3-part 1.5 [5], a simple criterion is presented, defining how the effects due to shear lag should be considered at serviceability and fatigue limit state verifications. The expressions for the evaluation of effective width under elastic and inelastic conditions are also determined. Shear lag should also be taken into account in composite steel and concrete structures, as it is stated in section 5.4 of EC4 [6-7]. However, the relevant guidelines do not specify the cases where shear lag affects significantly the global analysis. Contrary to the aforementioned materials, European regulations (EC2 [8]) do not examine the effect of shear lag in concrete structures. From all the above, it can be concluded that recommendations on shear lag effects in beams are based on a simplifying approach which may fail to capture satisfactorily the actual structural behavior of the member, since the influence of shear lag phenomenon is not constant along the beam length, while apart from the geometrical configuration of the cross section it depends also on the type of loading [9].
In up-to-date regulations, the important role of distortional effects in stress and strain distribution at beams is recognized. Nevertheless, relevant guidelines are often general, not proposing specific modelling methodologies for the aforementioned effects. Regarding the provisions of European Norms with respect to distortion of beams, it can be observed that most of the regulations of EC3-Part 1.1 [5] regarding torsion are valid only when distortional deformation can be neglected (section 6.2.7). The clauses referring to buckling resistance of members (section 6.3.3) are effective when distortional deformations do not appear at the cross section. The importance of distortion is also underlined in section 6.3.5.2 where it is stated that it should be prevented at the plastic hinge location. Distortional effect should be also taken into account during the design of unreinforced joints (Part 1.8 Section 7.5.2.1(7)) and evaluation of nominal stresses from fatigue actions (EC3, Part 1.9, Section 4 [5]). Regarding design of aluminum structures susceptible to fatigue, Eurocodes only mention that modified nominal stresses should be used in place of nominal stresses (EC9 Part 1.3 Section 5.2 [9]). Finally, apart from steel and aluminum structures, distortion also appears in concrete structures. Specifically, the opening of a joint in a box under certain specifications (EC2-Part 2-Section 6.3.2(106) [8]) may change the torsional resisting mechanism from Bredt circulatory torsion to a combination of warping torsion and St. Venant torsion. As a result the web shear due to torsion is practically doubled and significant distortion of the section takes place.

More attention to distortion matters is given in regulations for the design of steel and composite bridges. In particular, in Part 2 of EC3 [5] it is clearly stated that for members subjected to torsion, both torsional and distortional effects should be taken into account (6.2.7.1(1)) mentioning, nevertheless, that the aforementioned effects may be disregarded in the member where due to cross sectional transverse bending stiffness and/or diaphragm action, the effects from distortion do not exceed 10% of bending effects. Similar attention is paid in other guidelines as well such as those by American Association of State Highway and Transportation Officials (AASHTO) and Hanshin Expressway Public Corporation of Japan. According to these the maximum spacing of diaphragms placed in curved girder bridges is controlled so as to minimize distortional warping normal stresses in terms of bending normal ones (less than 10% for AASHTO, less than 5% for Hanshin Expressway Public Corporation of Japan) [11].

2. Higher Order Beam Theories taking into account In-Plane and Out-of-Plane Deformation

During the past years numerous research efforts have been published concerning the development of beam theories, including distortional effects. The vast majority of these research efforts focus on thin-walled profiles; hence, the simplifying assumptions of Thin Tube Theory TTT [12] play a dominant role for the formulation of relevant theories. Within the context of TTT, some of the first classical approaches tackle the torsional distortion problem by reducing it to an equivalent Beam-on-Elastic-Foundation (BEF) one [12-14]. This approach sheds light to some significant aspects concerning the development mechanism of distortion. Confirming BEF analogy, several research efforts expanded classical Vlasov TTT by formulating corresponding fourth-order differential equations including torsional distortion, introducing at the same time basic notions such as that of distortional warping, distortional center and distortional moment [11, 15-21]. The connection of the above mentioned additional distortional warping with secondary torsional shear stresses has been also highlighted [22]. Nevertheless, the previous studies focus mainly on box-shaped bridge decks and are not general, since they do not include flexural distortional effects, while restrictions are set with respect to cross sectional shape and its deformation (e.g. in-plane shear deformation of plate element of the beam due to distortion is neglected).
A major step towards generalizing Vlasov’s theory so as to incorporate flexural and torsional distortional effects is the development of GBT (Generalized Beam Theory). GBT was formulated by Schardt [23-25] and disseminated by Davies and co-researchers [26-29] through a series of publications studying linear static or buckling problems of thin-walled beams. The validity of these first GBT formulations is based on the assumption that the structural member has an open-shaped (initially un-branched) cross section which consists of thin rectangular walls/plates; thus thin plate theory assumptions are exploited for each wall. GBT analysis consists of two phases of analysis, namely a cross sectional analysis and a member analysis. Cross sectional analysis is then based on two stages which comprise the establishment of warping modes and the subsequent establishment of distortional ones by exploiting Vlasov’s zero-shear stress condition\(^1\). This procedure becomes more complicated in branched cross sections (i.e. in cross sections where a node may connect more than two plate elements) and in closed-shaped ones where zero-shear stress condition is not valid. Improvements towards addressing the above inherent disadvantages of the method have been systematically proposed and applied by Camotim, Silvestre and co-researchers in a series of publications expanding the method to cover a variety of cross sections, orthotropic materials, as well as geometrically nonlinear and inelastic problems [30-40]. A significant advantage of GBT stems from the general modal-decomposition-type character of its cross sectional analysis which has enabled researchers to attempt categorization of in-plane modes to global, distortional or local ones\(^2\). This categorization facilitates the comprehension of structural mechanisms and their hierarchical classification, confirming an inherent advantage of beam theories over more elaborate yet less insightful methods (e.g. shell or solid FEM models).

The number and accuracy of in-plane deformation modes computed during GBT cross sectional analysis are based on the “discretization” of the cross section to plate elements (to increase this number, intermediate nodes are added). This discretization is not a trivial task since it depends on the cross sectional shape and nodal topology (open-shaped branched or un-branched, closed-shaped branched or un-branched cross sections may be involved)\(^3\). Further developments of TTT formulations introduce the concept of eigenvalue-type cross sectional analysis [41-42] for the establishment of meaningful deformation modes together with the corresponding decay lengths of each mode, permitting their sorting in order of significance. Employing this concept some cumbersome procedures of GBT, as

\(^1\)A set of warping modes is obtained by applying unitary displacement at one node at the time while maintaining the rest equal to zero. The corresponding in-plane displacements of the segments are then determined for each of these modes based on the zero-shear condition along the mid-plane of each plate element set by Vlasov. This procedure leads to a loss of compatibility at natural nodes (i.e. at the corners of the plates). Hence, a rigid-body kinematic problem has to be solved to restore the displacement compatibility by applying normal displacements and rotations to each plate segment. Subsequently, force method is used to restore rotation compatibility at natural nodes, treating the plate segments as (deformable) continuous members and assuming their corners restrained by pinned supports [43].

\(^2\)According to these characterizations, which are usually present in buckling and post-buckling analysis of beams, rigid body translations/rotations of the cross sections already known from classical beam theories are considered as global modes. Local modes are separated from distortional ones, firstly through the observation that they do not include movements of natural nodes of the cross section but only transverse bending action of plate elements. Distortional modes, on the other hand, involve movements of these nodes combined with transverse bending action. Apart from this categorization, it should be also mentioned that within TTT/GBT, local modes do not activate warping displacements. Moreover, local modes are usually, but not necessarily, characterized by smaller decay lengths along beam length than distortional ones. Schafer and Ádány [49], employing Finite Strip Method, attempted to apply specific kinematic and strain constraints and classify in-plane deformation modes to the above groups, while they add an additional group of modes denoted as other. The latter group involves all these modes that do not fulfill the constraints set by Schafer and Ádány.

\(^3\)For example, an I-shaped cross section with stiffeners (branched cross section), following GBT cross sectional analysis procedure, contains multiple sets of nodes, namely natural dependent or independent nodes, end dependent or independent nodes and intermediate nodes. For further details the reader is referred to the review article of Camotim and team [50].
illustrated above, can be avoided. Eigenvalue cross sectional analysis has been employed by Ranzi and Luongo [43], Jönsson and Andreassen [44-47] and Vieira et al. [48] highlighting the fact that with this approach it is not necessary to classify cross sections according to their geometrical configuration (open-, close-shaped, branched or un-branched). Furthermore, the obtained modes contain the most significant ones and form a more meaningful and insightful basis of functions as compared to GBT ones which exhibit a more local character [43].

It is worth here noting that along with GBT, Finite Strip Method (FSM) has been also extensively applied for the analysis of beam-like thin-walled structural members. Similarly with GBT, FSM relies on plate theory assumptions which are applied on each wall of the cross section. FSM is more of a general purpose FEM-based method rather than a beam theory. Nevertheless, Schafer, Ádány and co-researchers have presented studies exploiting the advantages of FSM in combination with GBT in order to study buckling problems of beams. To this end, kinematical and strain constraints have been applied to conventional FSM leading the development of the so-called constrained FSM (cFSM) [49, 50-55]. cFSM uses specially selected constraints which enforce the member to deform according to mechanical constraints that match definitions of global or distortional buckling classes [54].

As mentioned in the previous paragraphs, all the above studies of the literature which incorporate distortional effects in beam analysis concern exclusively thin-walled profiles. Even though TTT offers the means for a simpler formulation, it restricts the range of application of relevant models, while it has been observed that its accuracy is questionable even in cases of cross sections that are classified as thin-walled (e.g. see the study of Tsipiras and Sapountzakis [56]). The problem of distortional analysis of beams of arbitrary cross section, exhibits increased complexity even though the basic analysis stages are the same with GBT ones. As far as cross sectional analysis is concerned, a suitable basis of out-of-plane deformation modes (warpings) accompanied by corresponding in-plane ones (distortions) has to be established as well; however this task exhibits far more complex behavior, since in the case of arbitrary cross section, kinematical considerations and constitutive relations cannot be simplified. Regarding member analysis, similarly with GBT, the formulation comprises the expression of displacement field as a linear combination of the obtained modes multiplied by relevant parameters which can be perceived as generalized coordinates. In this stage, the differences with TTT/GBT lie mainly on the use of dependent or independent parameters as generalized coordinates; thus, altering the handling of shear deformation⁴.

Towards solving the problem for arbitrarily shaped homogeneous or composite cross sections, St. Venant problem of prismatic elastic bodies plays a crucial role for the establishment of exploitable warping/distortional functions. This is due to the fact that in these problems the analysis of deformation reduces to the evaluation of 2-D functions over the cross sectional domain often referred to as central solutions (e.g. see the studies of Kosmatka and Dong [57] and Ie and Kosmatka [58]) for St. Venant solutions in anisotropic prismatic bodies). As it is widely known, in St. Venant problems the central solution is valid along the examined prismatic body provided that warping/in-plane deformations of the root cross section are not restrained (rigid body movements are though prohibited). However, central solution is usually applied in more general boundary conditions as suggested by Saint-Venant through the “principle of elastic

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⁴Usually, in GBT formulations shear deformation induced by each distortion/warping mode inserted in global analysis is not taken into account by using dependent parameters multiplying warping functions. In this case the first derivatives of the parameters multiplying the corresponding distortion modes are selected as dependent warping parameters. This consideration bears analogy with classic nonuniform torsion theory where primary torsional warping is multiplied by the rate of the angle of twist [76].
equivalence of statically equipollent systems of load” [59-61]. This statement establishes the so-called St. Venant’s Principle according to which the possible restraint near the support of the examined prismatic body does not affect the solution away from the support. However, towards expanding St. Venant’s theory, various researchers investigate the so-called end-effects (also referred to as extremity solutions, eigensolutions or transitional solutions [62]) when the above assumption is not maintained (e.g. in case of a clamped beam end). Hence, employing the semi-inverse method of St. Venant under static conditions, a displacement field of the following form can be assumed [61]

$$\bar{u}_{\text{SV}}(x, y, z) = \sum_{i=1}^{K} a_i(x) W_i(y, z)$$

(1b)

According to the above equations, the displacement vector $\bar{u}$ of an arbitrary point of the cross section is obtained as the sum of St. Venant solution vector $\bar{u}_{\text{SV}}$ (e.g. see Kosmatka and Dong [57]) combined with a residual displacement vector $\bar{u}_{\text{res}}$ due to end-effects which are responsible for the generation of self-equilibrating stress distributions [60-61, 63-64]. These additional displacements are written as a sum of $K$ two-dimensional functions $W_i$ (warping/in-plane deformation functions) multiplied by parameters $a_i$ expressing their longitudinal intensity. Stress components due to end-effects are considered to independently satisfy the equations of local equilibrium. Thus, applying the three-dimensional elasticity equilibrium equations, exploiting the exponentially decaying character of these end-effects and introducing a proper discretization scheme of the cross section, a 2-D quadratic eigenvalue problem is formulated. Hence, the concept of self-equilibrating end-effects permits the production of a basis of suitable modes of warping and in-plane deformation over the arbitrary cross sectional domain, by combining St. Venant 2-D central solution concept with eigenvalue analysis.

The formulation of advanced beam theories incorporating warping/distortional effects in beams of arbitrary cross sections has received limited amount of literature. In the majority of the relevant studies, the concepts described in previous paragraph have been used. Exploiting the insight offered by St. Venant solutions, El Fatmi and Ghazouani [65-67] employ St. Venant in-plane modes with corresponding warping ones for arbitrary orthotropic cross sections multiplied by independent parameters, in order to take into account their non-uniform distribution along the beam length. The same concept has been applied also by Petrov and Geradin [68] who formulated a theory for curved and pre-twisted beams of arbitrary homogeneous cross sections, covering geometrically nonlinear range as well. However, these formulations include only flexural and axial deformation modes originating from the influence of Poisson ratio, while torsional distortion is excluded. Hence, it could be stated that these research efforts, study Poisson ratio effects rather than distortion effects. The concept of eigenvalue analysis has been also exploited in some recent research efforts. More specifically, Ferradi and Cespedes [69] formulate a beam element performing FEM-based eigenvalue analysis concerning the distortional behavior of the cross section (in-plane problem) and compute warping functions separately by employing an expanded equilibrium scheme based on a previous study of the same authors [70]. Pai
[62] presented a FEM formulation for cross sectional analysis introducing the concept of zero warping-induced work. This analysis is employed to study isotropic or anisotropic beams undergoing large deformation. Additionally, Genoese et al. [71] developed a FEM procedure based on a mixed variational formulation for orthotropic beams by developing an eigenvalue cross sectional problem yielding simultaneously distortional and warping functions of the arbitrarily-shaped cross section. Finally, Dikaros and Sapountzakis [72] developed an advanced beam element in which the so-called sequential equilibrium scheme is employed for the computation of warping and distortional functions. The advantages of this approach over the eigenvalue analysis one are that distortional and warping functions are evaluated by the same problem and in order of significance, while contrary to eigenvalue analysis, it permits the separate evaluation of axial, flexural and torsional mode groups5. Furthermore, in this way, a beam formulation is developed employing a relatively small number of unknown functions by employing the most “significant” modes (first solutions of the sequential equilibrium scheme) in order to keep the simplicity of the formulation to the highest possible level.

Due to the fact that axial mode is crucial for buckling (and generally non-linear) analysis, the advancements regarding higher order beam theories taking into account axial warping (out-of-plane deformation) and axial distortional (in-plane deformation) modes are worth mentioning. Petrov and Géradin [68] developed a theory for curved and twisted beams where an axial distortional mode is included. El Fatmi and Ghazouani [67, 73] developed a higher order composite beam theory built on Saint-Venant’s solution taking into account an axial distortional mode. Jönsson and Andreassen [44] as well as Ranzi and Luongo [43] calculated warping and distortional modes solving eigenvalue problems in the context of the Generalized Beam Theory (GBT), i.e. “a thin-walled prismatic bar theory that includes cross-section in-plane and out-of-plane (warping) deformation through the consideration of so-called cross-section deformation modes” [74]. Jang et al. [75] analyzed thin-walled straight beams with generally shaped closed sections using numerically determined sectional deformation functions (warping and distortional modes are obtained through an eigenvalue problem). Gonçalves and Camotim [77] studied elastic buckling of uniformly compressed thin-walled regular polygonal tubes in the concept of GBT, including axial warping and distortional modes. Genoese et al. [71] presented a generalized model for heterogeneous and anisotropic beams including section distortions, integrating axial warping and distortional modes for both thin and thick walled cross-sections through the solution of an eigenvalue problem. Sapountzakis et al. [78] presented an advanced beam element under longitudinal external loading by BEM where the first axial warping mode is evaluated with respect to external loading. Finally, Ferradi et al. [79] developed a model reduction technique for beam analysis with the asymptotic expansion method where the warping and distortional deformation modes are determined, in function of the applied loads and the limit conditions of the problem.

However, the majority of the aforementioned beam theories cannot be generally applied in any type of cross-section

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5Eigenvalue analysis, by sorting the modes in descending order of importance, permits the comparative study of the significance of (axial, flexural, torsional) warping/distortion modes. Hence, light is shed according to the overall structural behavior of the member (e.g. open members are more susceptible to torsional deformations than flexural ones). Nevertheless, this approach does not permit the versatile formulation of the corresponding beam theory. In order to keep the formulation simple, inspection is required by the analyst so as to select manually the most significant axial, flexural and torsional modes. On the other hand, selecting a pre-defined number of modes would increase unnecessarily the number of involved beam d.o.f.s and hence the complexity of relevant software. These drawbacks inhibit the main goal of this doctoral thesis which is to develop a general purpose, versatile yet accurate computational tool for the analysis of framed structures.
or loading condition. More specifically, in [67-68, 73] only the first axial distortional mode is employed, in [78] only the first axial warping mode is employed, while in [43-44, 75-77] assumptions of thin-walled beam theory are adopted. Moreover, in [79] warping functions are calculated in function of applied loads and limit conditions, requiring a new cross sectional analysis for each loading or limit condition, which is outside the scope of a higher order beam theory. Additionally, in [71], where the cross-section can be thin- or thick- walled and assumptions of thin walled beam theory are not adopted, axial modes are calculated through the solution of an eigenvalue problem. Finally Argyridi and Sapountzakis [80] developed a higher order beam theory where axial warping and distortional modes are evaluated employing the concept of sequential equilibrium scheme. In table 1 warping and distortional modes of a hollow rectangular cross-section are presented according to the sequential equilibrium scheme.

3. Higher Order Beam Theories taking into account In-Plane and Out-of-Plane Deformation in Buckling Analysis

Elastic stability of beams is one of the most important criteria in the design of structures. Thus, in this paper the literature overview regarding higher order beam theories is presented as it is considered of high interest. Chen and co-workers [81] were the first that included a simple analytical model in their beam formulation to account for the effects of local buckling of circular cross-section. Since then numerous of research efforts have been published concerning buckling including shear lag and distortional effects in the concept of a beam theory. Some researchers, have studied local buckling of beams employing Generalized Beam Theory (GBT) i.e. “a thin-walled prismatic bar theory that includes cross-section in-plane and out-of-plane (warping) deformation through the consideration of so-called cross-section deformation modes” [74]. Davies and coworkers used GBT to investigate the buckling of cold-formed steel (open-section) profiles [82], while Camotim and co-workers studied local buckling of beams regarding steel and aluminum columns [31], thin walled regular polygon tubes, angle, T-sections and cruciform thin-walled members [35, 77, 84], cold formed steel purlins [85], steel-concrete composite beams [74], generally loaded thin-walled members with arbitrary flat-walled cross-sections [83] employing GBT. Other researchers studied buckling problems of beams employing Finite Strip Method (FSM) [52-53, 86] that relies on plate theory assumptions which are applied on each wall of the cross-section. Karamanos and co-workers formulated a model that accounts for global behavior and for local buckling as well. The global (beam type) response is described through Lagrange polynomials and the cross sectional ovalization / warping in terms of trigonometric functions. This formulation has been quite successful in simulating local buckling in circular hollow section members [87-90]. Local buckling has also been examined for several cases of thin walled sections [e.g. 91-94]. All the aforementioned researches deal with the problem of local buckling, however they employ assumptions of Thin Tube Theory and in some cases their application is limited by the cross-sectional shape. For this purpose, Argyridi and Sapountzakis [80] developed a higher order beam theory for the buckling analysis of arbitrarily shaped beams where warping and distortional modes (axial additionally to flexural and torsional ones) are evaluated employing the concept of sequential equilibrium scheme.
Table 1. Warping and distortional modes of a hollow rectangular cross section according to the sequential equilibrium scheme.

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<th>Warping Modes</th>
<th>Distortional Modes</th>
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<td><strong>1&lt;sup&gt;st&lt;/sup&gt; Sequential Equilibrium Stage</strong></td>
<td><strong>2&lt;sup&gt;nd&lt;/sup&gt; Sequential Equilibrium Stage</strong></td>
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<td>Flexural Modes for Flexure about Strong Axis</td>
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<td>Flexural Modes for Flexure about Weak Axis</td>
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<td>Torsional Modes</td>
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Min value | Max value
4. Conclusion
In this paper, a brief literature overview is presented regarding the major beam theories taking account out-of-plane (warping) and in-plane (distortional) deformation. The analysis of a member employing a beam theory consists of two parts. In the first part, a cross-sectional analysis is conducted calculating the geometric constants of the cross-section of the beam. In the second part, the aforementioned geometric constants are substituted in the global equilibrium equations of the beam to calculate the response of the member under any kind of loading. The literature overview depicted that the main difference among the various higher order beam theories is the way that the cross-sectional characteristics are evaluated.

5. Conflicts of Interest
The author(s) report(s) no conflict(s) of interest(s). The author along are responsible for content and writing of the paper.

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